Goniometria & trigonometria

INFOMAAT
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Formule base

Gradi vs Radianti	Radianti vs Gradi	Prima relazione fondamentale
$\alpha^\circ = \alpha^r \cdot \frac{180^\circ}{\pi}$	$\alpha^r = \alpha^\circ \cdot \frac{\pi}{180^\circ}$	$\sin^2\alpha + \cos^2\alpha = 1$

Seconda relazione fondamentale	Relazione tangente - cotangente
$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$	$\cot\alpha = \frac{1}{\tan\alpha} = \frac{\cos\alpha}{\sin\alpha}$
Cosecante	**Secante**
$\csc\alpha = \frac{1}{\sin\alpha}$	$\sec\alpha = \frac{1}{\cos\alpha}$

Archi associati

$\sin(-\alpha) = -\sin\alpha$	$\cos(-\alpha) = \cos\alpha$	$\tan(-\alpha) = -\tan\alpha$
$\sin\left(\frac{\pi}{2} \pm \alpha\right) = \cos\alpha$	$\cos\left(\frac{\pi}{2} \pm \alpha\right) = \mp\sin\alpha$	$\tan\left(\frac{\pi}{2} \pm \alpha\right) = \mp\cot\alpha$
$\sin(\pi \pm \alpha) = \mp\sin\alpha$	$\cos(\pi \pm \alpha) = -\cos\alpha$	$\tan(\pi \pm \alpha) = \pm\tan\alpha$
$\sin(2\pi \pm \alpha) = \pm\sin\alpha$	$\cos(2\pi \pm \alpha) = \cos\alpha$	$\tan(2\pi \pm \alpha) = \pm\tan\alpha$

Funzioni inverse

$\sin(\arcsin\alpha) = \alpha$	$\cos(\arccos\alpha) = \alpha$
$\sin(\arccos\alpha) = \sqrt{1-\alpha^2}$	$\cos(\arcsin\alpha) = \sqrt{1-\alpha^2}$
$\sin(\arctan\alpha) = \frac{\alpha}{\sqrt{1+\alpha^2}}$	$\cos(\arctan\alpha) = \frac{1}{\sqrt{1+\alpha^2}}$
$\tan(\arcsin\alpha) = \frac{\alpha}{\sqrt{1-\alpha^2}}$	$\tan(\arccos\alpha) = \frac{\sqrt{1-\alpha^2}}{\alpha}$

	$\sin\alpha$	$\cos\alpha$	$\tan\alpha$	$\cot\alpha$
$0°$	0	1	0	✗
$30° = \frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
$45° = \frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
$60° = \frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
$90° = \frac{\pi}{2}$	1	0	✗	0
$180° = \pi$	0	-1	0	✗
$270° = \frac{3\pi}{2}$	-1	0	✗	0
$360° = 2\pi$	0	1	0	✗

Formule addizione

Formule addizione- sottrazione coseno

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

Formule addizione- sottrazione seno

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

Formule addizione- sottrazione tangente

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Formule duplicazione

$\sin 2\alpha = 2\sin\alpha\cos\alpha$	$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$	$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$

Formule linearizzazione

$\sin^2\alpha = \frac{1-\cos 2\alpha}{2}$	$\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$	$\tan^2\alpha = \frac{1-\cos 2\alpha}{1+\cos 2\alpha}$

Formule bisezione

$\sin\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{2}}$	$\cos\frac{\alpha}{2}=\pm\sqrt{\frac{1+\cos\alpha}{2}}$	
$\tan\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$	$\tan\frac{\alpha}{2}=\frac{1-\cos\alpha}{\sin\alpha}$	$\tan\frac{\alpha}{2}=\frac{\sin\alpha}{1+\cos\alpha}$

Formule parametriche

$\sin x=\frac{2t}{1+t^2}$	$\cos x=\frac{1-t^2}{1+t^2}$	$\tan x=\frac{2t}{1-t^2}$ $t=\tan\alpha/2$

Formule di prostaferesi

$$\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha-\sin\beta=2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$

$$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

Formule di Werner

$$\sin\alpha\cos\beta=\frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$$

$$\cos\alpha\cos\beta=\frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$$

$$\sin\alpha\sin\beta=\frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$$

Equazioni goniometriche con identità

$\sin f(x) = \sin g(x)$	$\begin{cases} f(x) = g(x) + 2k\pi \\ f(x) + g(x) = \pi + 2k\pi \end{cases}$
$\cos f(x) = \cos g(x)$	$f(x) = \pm g(x) + 2k\pi$
$\tan f(x) = \tan g(x)$	$f(x) = g(x) + k\pi$

$$\begin{bmatrix} -\sin f(x) = \sin(-f(x)) \\ \cos f(x) = \sin(\pi/2 - f(x)) \\ -\cos f(x) = \cos(\pi - f(x)) \\ -\tan f(x) = \tan(-f(x)) \end{bmatrix}$$

Metodo angolo aggiunto

$$f(x) = a\sin x + b\cos x + c$$

$$f(x) = A\sin(x + \phi) + c$$

$A = \sqrt{a^2 + b^2}$	$\begin{cases} \cos\phi = \dfrac{a}{\sqrt{a^2 + b^2}} \\ \sin\phi = \dfrac{b}{\sqrt{a^2 + b^2}} \end{cases}$

Goniometria e geometria analitica

Angolo formato tra due rette

$$\arctan\left|\frac{m - m'}{1 + mm'}\right|$$

Circonferenza di centro $C(x_0, y_0)$ e raggio r

$$\begin{cases} x = x_0 + r\cos\alpha \\ y = y_0 + r\sin\alpha \end{cases} \quad \text{CON } \alpha \in [0, 2\pi]$$

Ellisse di centro $C(x_0, y_0)$, assi paralleli agli assi cartesiani e semiassi di ,misure a e b

$$\begin{cases} x = x_0 + a\cos\alpha \\ y = y_0 + b\sin\alpha \end{cases}$$